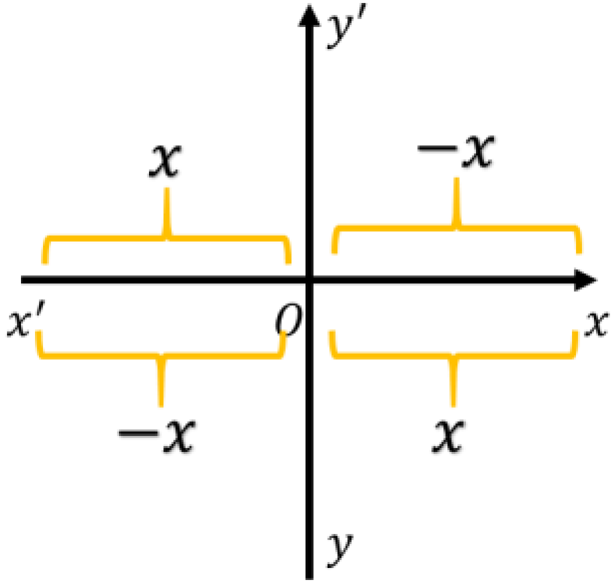
Functions

Limits (part 4)

Properties of limit

- If $\lim_{x \to a} f(x)$ exist, then this limit is unique.
- $\lim_{x \to -\infty} f(x) = \lim_{x \to +\infty} f(-x)$



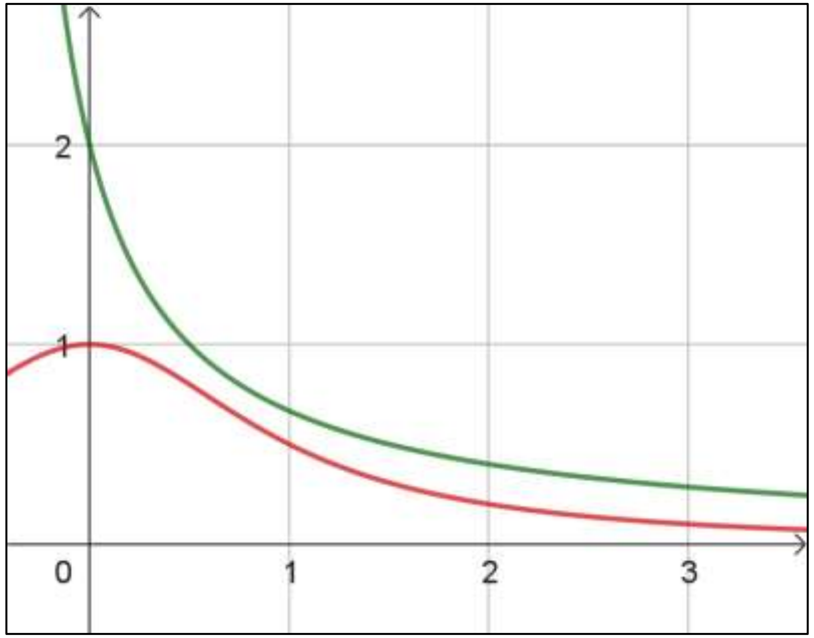
Properties of limit

Note that if $f(x) < g(x)$

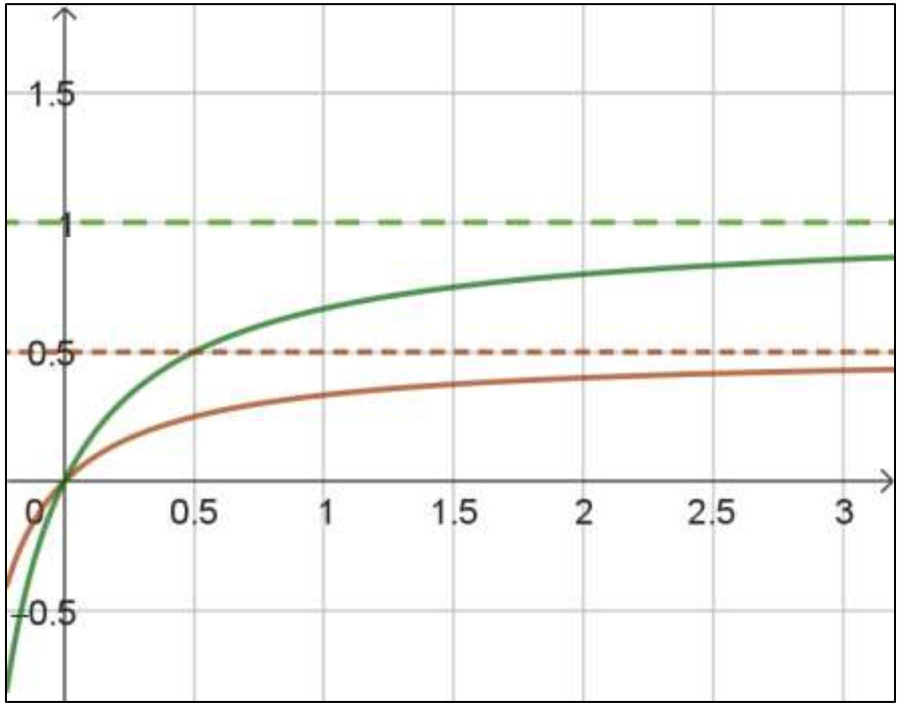
So the curve of f is below the curve of g.

- If $f(x) \leq g(x)$, then $\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} g(x) = 0$$

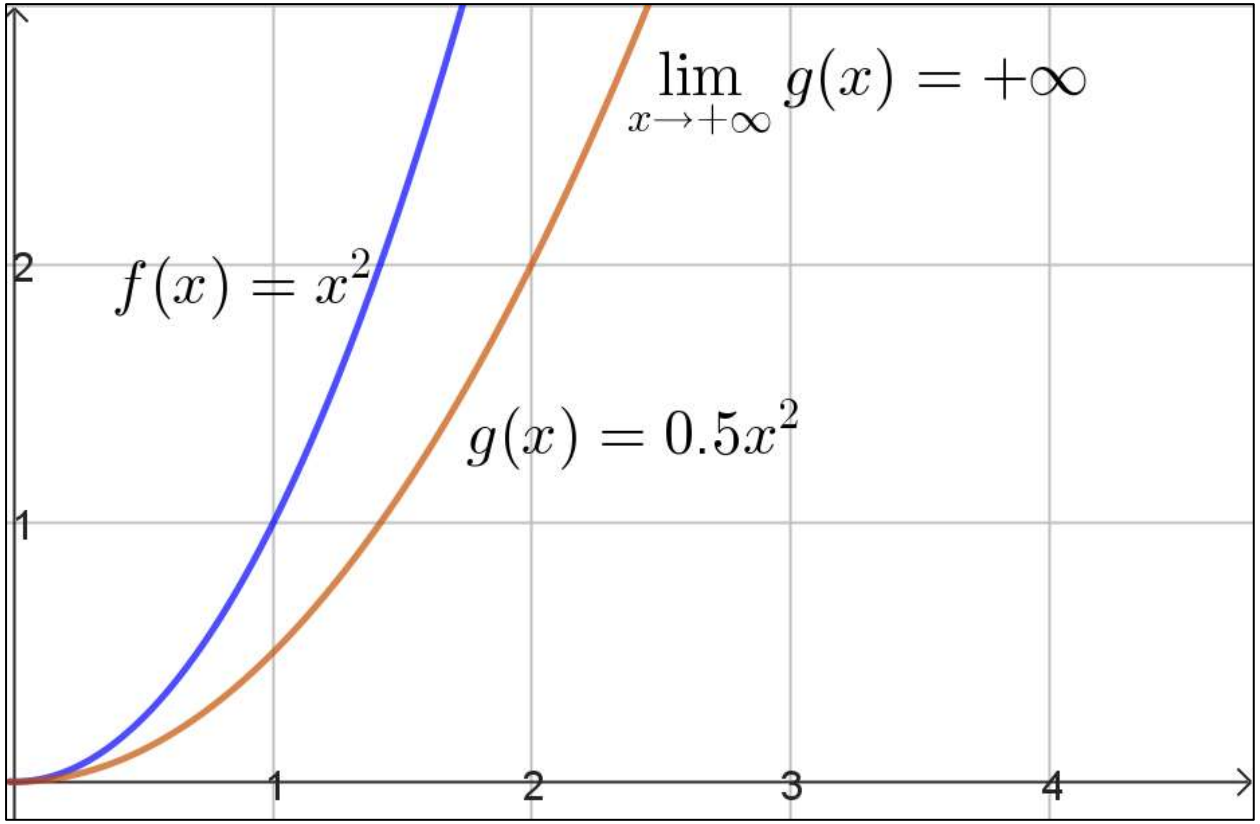


$$\lim_{x \to +\infty} f(x) > \lim_{x \to +\infty} g(x)$$



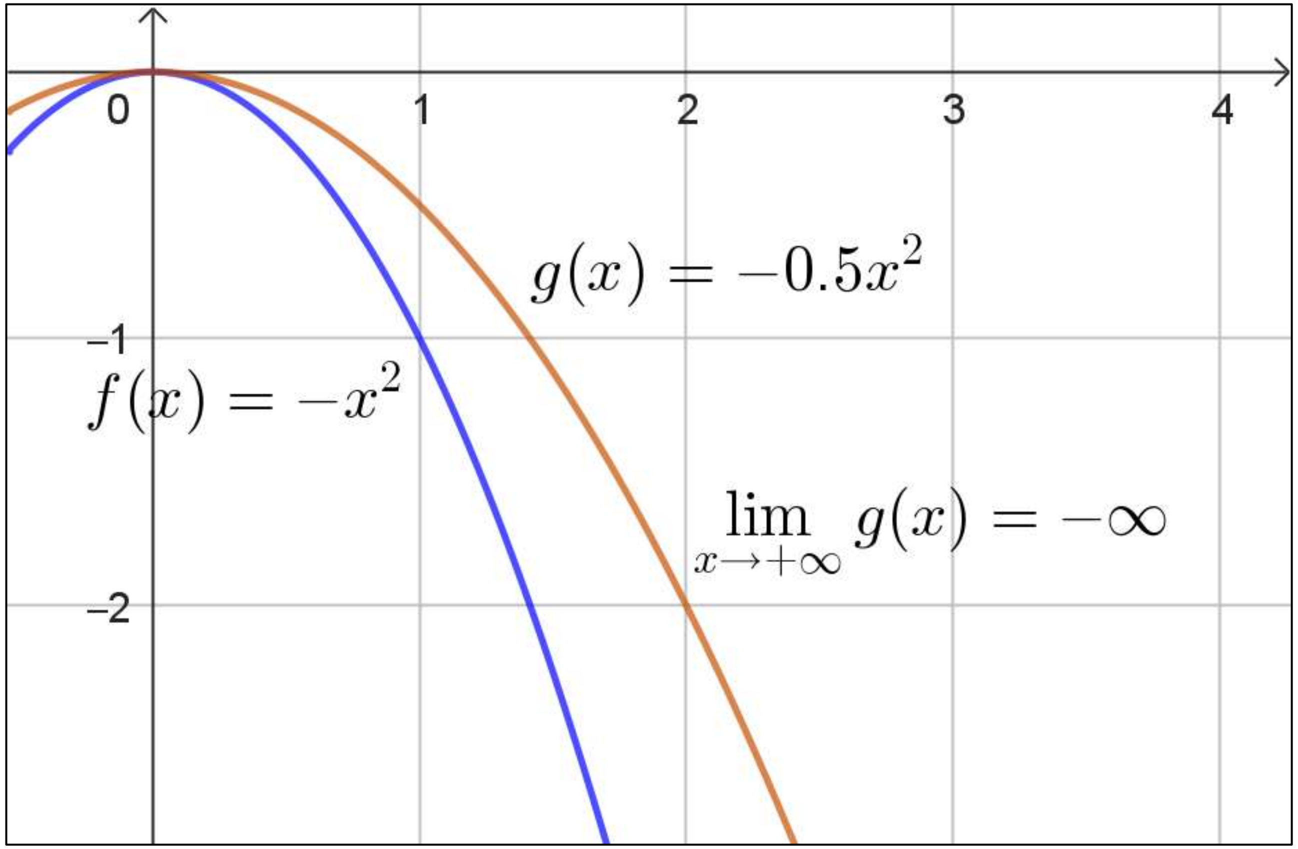
Properties of limit

❖ If $f(x) \geq g(x)$ and $\lim_{x \to a} g(x) = +\infty$, then $\lim_{x \to a} f(x) = +\infty$



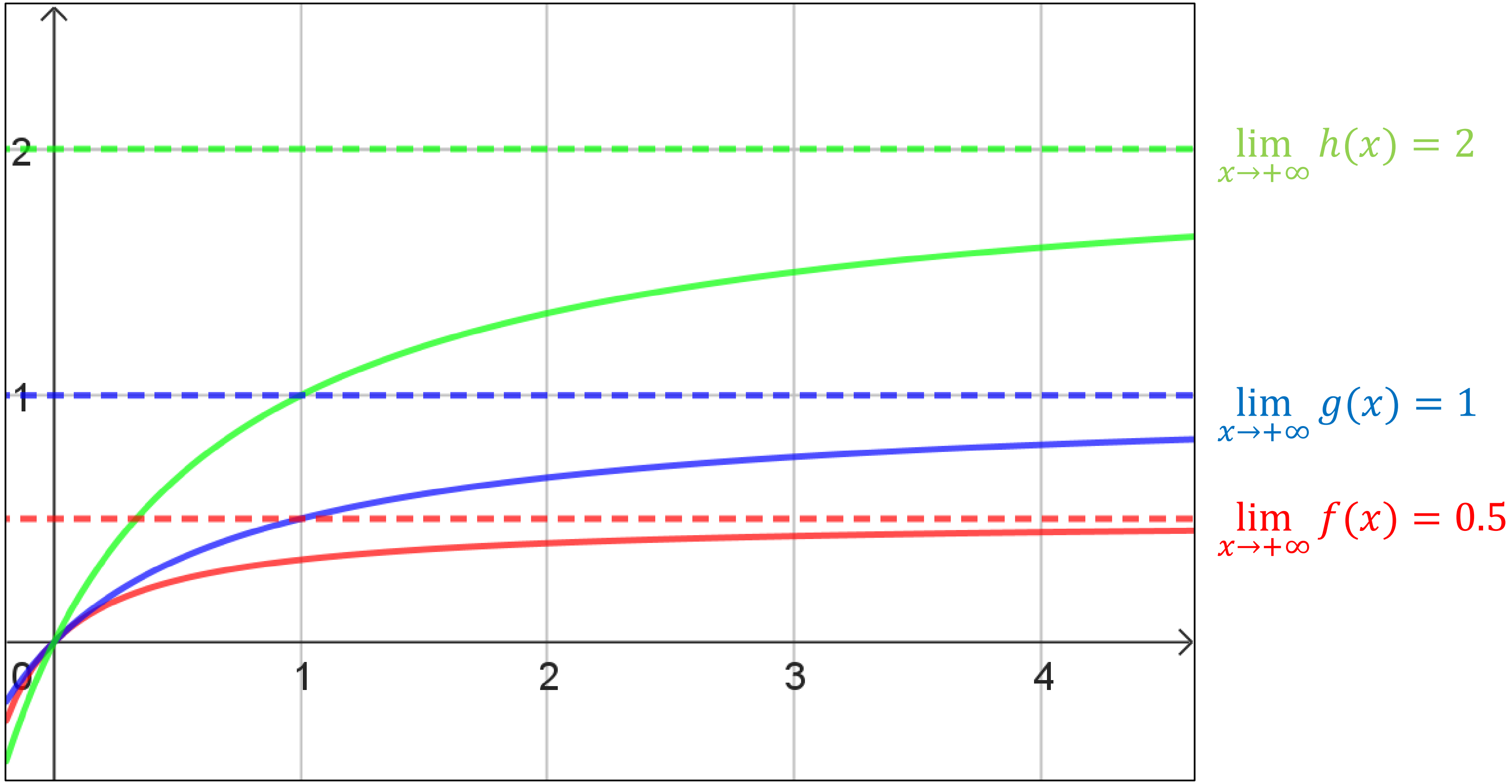
Properties of limit

❖ If $f(x) \leq g(x)$ and $\lim_{x \to a} g(x) = -\infty$, then $\lim_{x \to a} f(x) = -\infty$



Properties of limit

❖ If $f(x) \leq g(x) \leq h(x)$, then $\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x) \leq \lim_{x \to a} h(x)$

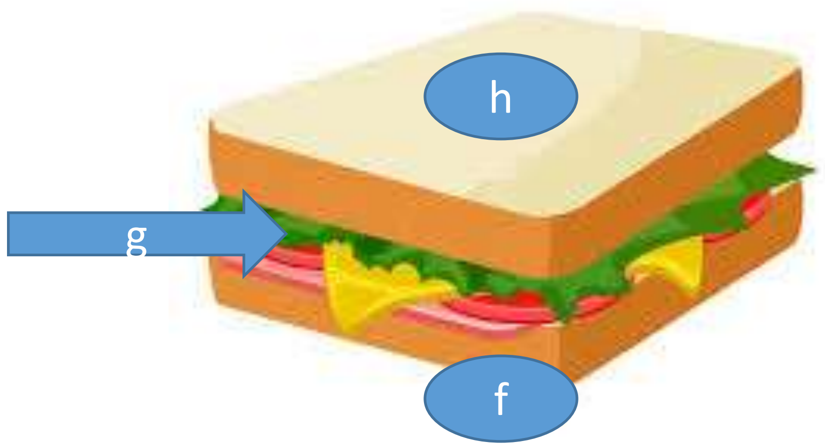


Squeeze theorem (Sandwich theorem)

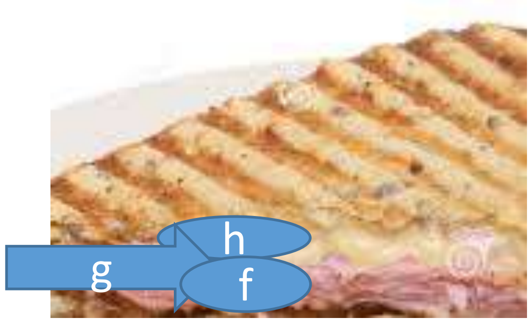
If $f \leq g \leq h$ &

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = m \quad (where\ m \in IR\ or\ \ m = +\infty\ \ or\ -\infty)$$

Then $\lim_{x \to a} g(x) = m$



After the Sab



Lim f=lim g=lim h

Squeeze theorem (Sandwich theorem) Example 1

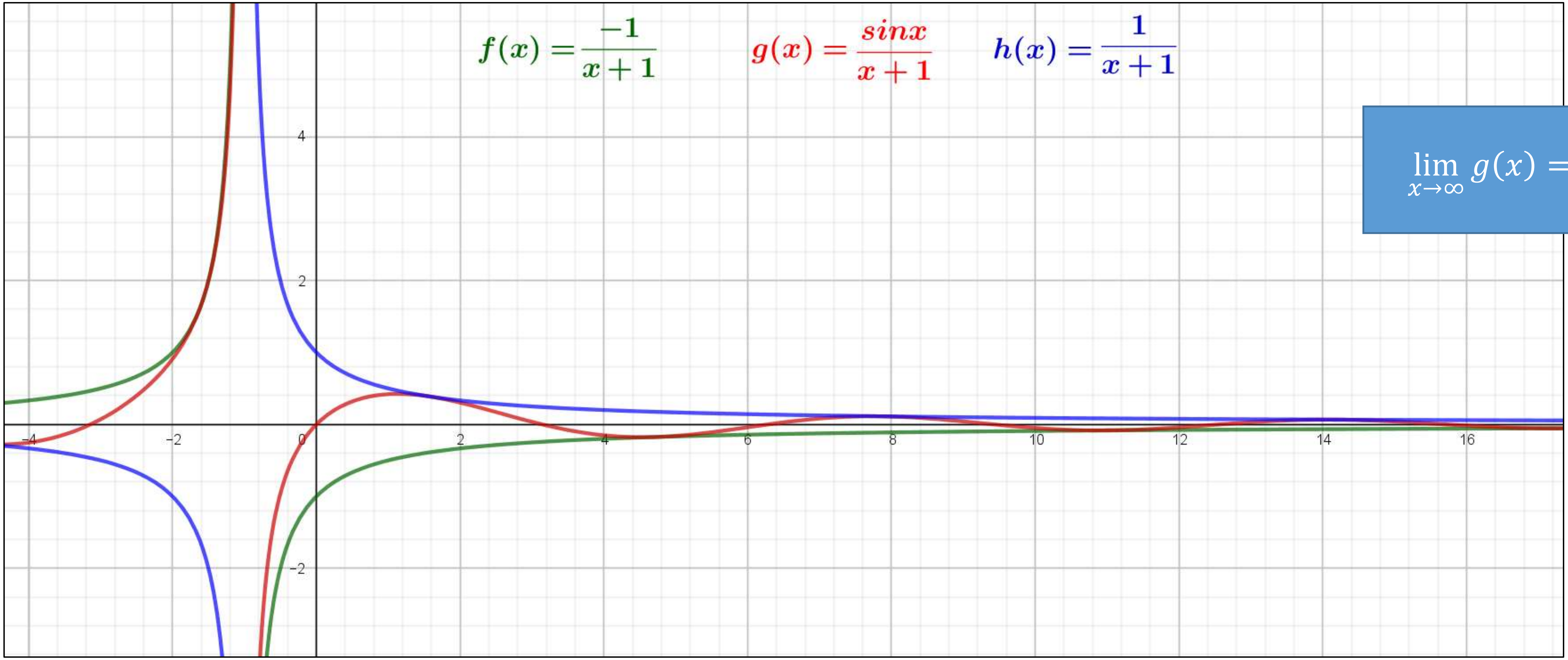
$$\frac{-1}{x+1} \leq \frac{sinx}{x+1} \leq \frac{1}{x+1}$$

Then $\lim_{x\to\infty} \frac{-1}{x+1} \leq \lim_{x\to\infty} \frac{sinx}{x+1} \leq \lim_{x\to\infty} \frac{1}{x+1}$

$\lim_{x\to\infty} \frac{-1}{x+1} = 0$ & $\lim_{x\to\infty} \frac{1}{x+1} = 0$

Hence , $\lim_{x\to\infty} \frac{sinx}{x+1} = 0$

Squeeze theorem (Sandwich theorem) Example 1



$$\lim_{x \to \infty} g(x) = 0$$

Squeeze theorem (Sandwich theorem) Example 2

Find $\lim_{x \to +\infty} \frac{x + \cos x}{x}$

$$\frac{x-1}{x} \le \frac{x+cosx}{x} \le \frac{x+1}{x}$$

$$\frac{x}{x} - \frac{1}{x} \le \frac{x+cosx}{x} \le \frac{x}{x} + \frac{1}{x}$$

$$1 - \frac{1}{x} \le \frac{x+cosx}{x} \le 1 + \frac{1}{x}$$

Squeeze theorem (Sandwich theorem) Example 2

$$\lim_{x \to +\infty}\left(1-\frac{1}{x}\right) \leq \lim_{x \to +\infty}\frac{x+cosx}{x} \leq \lim_{x \to +\infty}\left(1+\frac{1}{x}\right)$$

$$\lim_{x \to +\infty}\left(1-\frac{1}{x}\right) = \lim_{x \to +\infty}\left(1+\frac{1}{x}\right) = 1$$

Sandwich Theorem

$$\lim_{x \to +\infty}\left(\frac{x+\cos x}{x}\right) = 1$$